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On the Observability and Controllability of Active Suspension System

Omorodion Ikponwosa Ignatius
Faculty of Engineering and Technology, Ambrose Alli University, PO box 14, Ekpoma, Edo State. Nigeria

Oviawe C.I
Department of Mechanical Engineering
Edo State Institute of Technology and Management Usen, Edo State, Nigeria

Abstract
The focus of this paper is on the controllability and observability of an active suspension system used in automobile. The primary responsibility of control system Engineers is to design and implement controller. The active suspension system dynamics was captured by a mathematical model. The system transfer function model was determined by using the road disturbance as input and the car response as output. The state – space representation was subjected to controllability and observability test using MATLAB commands. The result of the test shows that the rank of the state matrix was (4) which is equal to the state matrix dimension. The active suspension system is both state controllable and observable.
Keywords: Controllability, Observability, Active Suspension.

1. Introduction

Suspension system can be classified into; passive, semi-active and active suspension system. Traditional suspension consists springs and dampers are referred to as passive suspension, then if the suspension is externally controlled it is known as a semi active or active suspension. An early design for automobile suspension systems was focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability Alleyen and Hedrick, (1995). Thus the passive suspension system, which approach optimal characteristics had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, the active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, and wheel velocity and wheel and body acceleration Esmailzadeh and Taghirad, (1997). An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. The focus of this thesis is on active suspension system controller design. "The process of selecting controller parameters to meet given performance specifications is known as controller tuning" (Ogata, 2002, p. 682). A variety of theoretical approaches have been used to produce PID-tuning formulas for a first-order plant with time delay. A heuristic time-domain analysis (Hang et al., 1991) used set-point weighting to improve Ziegler and Nichols' (1942) original PID-tuning formulas, which were also determined empirically. "Repeated optimizations using a third-order Padé approximation of time delay produced tuning formulas for discrete values of normalized dead time" (Zhuang and Atherton, 1993). Barnes et al., (1993) used open-loop frequency response to design PID controllers by finding the least – squares fit between the desired Nyquist curve and the actual curve. In reviews of the performance and robustness of both PI- and PID-tuning formulas, tuning algorithms optimized for set point change response were found to have a gain margin of around 6 dB, and those that optimized for load disturbance had margins of around 3.5 dB (Ho et al., 1995; Ho et al., 1996). PID-tuning formulas were derived by identifying closed-loop pole positions on the imaginary axis, yielding the system's ultimate gain and period. Dynamics are said to suffer, however, for processes where time delay dominates "due to the existence of many closed-loop poles near the imaginary axis, where the effect of zero addition by the derivative term is insignificant to change the response characteristics" (Mann et al., 2001). The concept of controllability and observability were introduced by Kalman and they play an important role in the design of control system in state space. If a process is controllable it means all the installed actuators excite all the structural modes of the system. And if the system is observable then it means that the installed sensors detect the motions of all the modes. A process is said to be completely controllable if it can be transferred from any initial state to any desired state in a finite time interval by some unconstrained control. In other words, the process is transferable if it is possible to find a control vector $u(t)$ which, in specified time t_f, will transfer the process between two arbitrarily specified finite states x_o and x_f. A process is said to be completely observable if every state $x_o(t_o)$ can be completely identified by measurement of the output over a finite time interval. If a process is not completely observable it means that some of the state variables are shielded from observation. In other words a process is completely observable if any initial state can be determined by observing the output

2. Mathematical Modelling of Automobile Active Suspension for Quarter Car Model

Designing an automobile suspension system to meet performance specification of ride comforts and good road handling is an interesting and challenging control problem. For ease of design, analysis and simulation, quarter automobile suspension system model is used to simplify the problem. This model represent an active suspension where an actuator is included that is able to generate the required control force to control the automobile dynamics. From the quarter car model, the design can be extended into full car model.

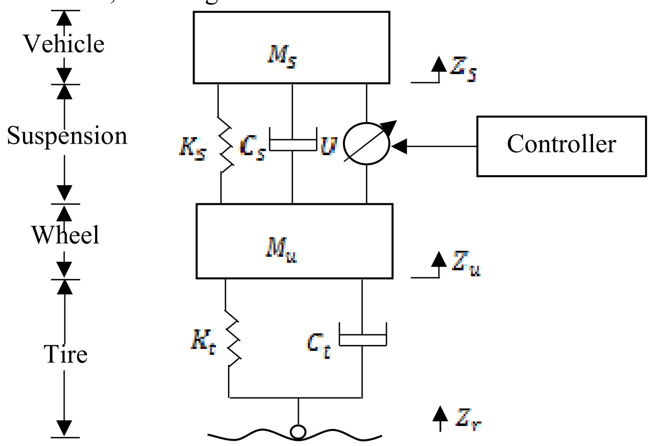

Figure 1: Quarter Car Model

Figure 1 above shows a basic two – degree – of freedom system representing the model of a quarter car, where the quarter mass of the automobile is represented with M_S, referred to as sprung mass in control and dynamics literature. M_U is the mass of the wheel (unsprung mass). The spring constant of the suspension is K_S, K_t is spring constant of the wheel and tire respectively. The damping constant of the suspension system is C_s while C_t is the damping constant of the wheel and tire respectively. U is the control force of the actuator. From the figure 1 above and using the Newton's law, we can obtain the system equations of motion below:

For M_S,
F = Ma,

$$M_s\ddot{Z}_s(t) = C_s[\dot{Z}_u(t) - \dot{Z}_s(t)] + K_s[Z_u(t) - Z_s(t)] + U(t) \tag{1}$$

For M_U,
F = Ma,

$$M_u\ddot{Z}_u(t) = C_s[\dot{Z}_s(t) - \dot{Z}_u(t)] + K_s[Z_s(t) - Z_u(t)] + K_t[Z_r(t) - Z_u(t)] + C_t[\dot{Z}_r(t) - \dot{Z}_u(t)] + U(t) \tag{2}$$

where

$Z_S - Z_U =$ Suspension travel

$\dot{Z}_S =$ Car body Velocity

$\ddot{Z}_S =$ Car Body Acceleration

$Z_U - Z_r =$ Wheel Deflection

$\dot{Z}_U =$ Wheel Velocity

$Z_U =$ Wheel Acceleration

In control theory, the transfer function of a system is defined in terms of an output to input ratio, but the use of a transfer function in system dynamics and vibration testing implies certain physical properties, depending on whether position, velocity, or acceleration is considered as the response (output). From equation (1) and (2);

taking Z_S and Z_U as output variables and taking U and Z_r as input variable respectively. The system transfer function is shown below.

$$G_1(s) = \frac{Z_S(s) - Z_u(s)}{U(s)} = \frac{(M_S + M_u)S^2 - C_t S + K_t}{\Delta} \quad (3)$$

$$G_2(s) = \frac{Z_S(s) - Z_u(s)}{Z_r(s)} = \frac{M_S C_t S^3 - M_u K_t S^2}{\Delta} \quad (4)$$

where

$$\Delta = (M_S S^2 + C_S S + K_S)\cdot(M_u S^2 + (C_S + C_t)S + (K_S + K_t) - (C_S S + K_S)\cdot \quad (C_S S + K_S) \quad (5)$$

4. Observability and Controllability

Using pole placement, it is not always possible to find a control law of a given form that causes the eigenvalues of the closed – loop system to have desired values. This variability to find a suitable control law raises the concept of controllability. A system is said to be completely controllable or state controllable if every state variable (i.e. all positions and velocities) can be affected in such a way, as to cause it to reach a particular value within a finite amount of time by some unconstrained (unbounded) control, $U(t)$. If one state variable cannot be affected in this way, the system is said to be uncontrollable. A similar concept to controllability is the idea that every state variable in the system has some effect on the output of the system (response) and is called observability. A system is observable if examination of the response (system output) determines information about each of the state variables.

4.1 State Space Representation

The n – dimensional space whose coordinate axes consists of the X_1 – axis, X_2 – axis . . . X_n is called state space. Any state can be represented by a point in the state space.

$$\dot{X}(t) = AX(t) + BU(t) \quad (5)$$

$$Y(t) = CX(t) + DU(t) \quad (6)$$

These equations are vector differential equations where $'x'$ is the n – dimensional state vector.
For the automobile suspension system, using MATLAB the system transfer function is transformed into the state space form shown below.

$$\begin{bmatrix} \dot{X}_1 \\ \ddot{X}_1 \\ \dot{Y}_1 \\ \ddot{Y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-C_S C_t}{M_S M_t} & 0 & \left[\frac{C_S}{M_S}\left(\frac{C_S}{M_S} + \frac{C_S}{M_u} + \frac{C_t}{M_u}\right) - \frac{K_S}{M_S}\right] & \frac{-C_S}{M_S} \\ \frac{C_t}{M_u} & 0 & -\left(\frac{C_S}{M_S} + \frac{C_S}{M_u} + \frac{C_t}{M_u}\right) & 1 \\ \frac{K_t}{M_u} & 0 & -\left(\frac{K_S}{M_S} + \frac{K_S}{M_t} + \frac{K_t}{M_u}\right) & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ Y_1 \\ \dot{Y}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_S} & \frac{C_S C_t}{M_S M_u} \\ 0 & \frac{-C_t}{M_u} \\ \left(\frac{1}{M_S} + \frac{1}{M_u}\right) & \frac{-K_t}{M_u} \end{bmatrix} \begin{bmatrix} U \\ Z_r \end{bmatrix} \quad (7)$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ Y_1 \\ \dot{Y}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U \\ Z_r \end{bmatrix} \quad (8)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-C_S C_t}{M_S M_t} & 0 & \left[\frac{C_S}{M_S}\left(\frac{C_S}{M_S} + \frac{C_S}{M_u} + \frac{C_t}{M_u}\right) - \frac{K_S}{M_S}\right] & \frac{-C_S}{M_S} \\ \frac{C_t}{M_u} & 0 & -\left(\frac{C_S}{M_S} + \frac{C_S}{M_u} + \frac{C_t}{M_u}\right) & 1 \\ \frac{K_t}{M_u} & 0 & -\left(\frac{K_S}{M_S} + \frac{K_S}{M_t} + \frac{K_t}{M_u}\right) & 0 \end{bmatrix} \text{ system matrix} \quad (9)$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{1}{M_S} & \frac{C_S C_t}{M_S M_u} \\ 0 & \frac{-C_t}{M_u} \\ \left(\frac{1}{M_S}+\frac{1}{M_u}\right) & \frac{-K_t}{M_u} \end{bmatrix}$$ Control Input Matrix (10)

$C = [0 \ \ 0 \ \ 1 \ \ 0]$ Output Matrix (11)

$D = [0 \ \ 0]$ Direct Transmission Matrix (12)

$$X(t) = \begin{bmatrix} X_1 \\ \dot{X}_1 \\ Y_1 \\ \dot{Y}_1 \end{bmatrix}$$ State Vector (13)

$$U(t) = \begin{bmatrix} U \\ Z_r \end{bmatrix} =$$ Input Vector (14)

$Y(t) =$ Output vector

4.2 Controllability Matrix

A linear time invariant continuous system described by the equation:

$$\dot{x} = Ax + Bu \quad (15)$$

$$y = Cx + Du \quad (16)$$

Is completely controllable if and only if the rank of the controllability matrix is defined as:

$$Qc = [B : AB : \ A^2B : \dots \dots A^{n-1}B] \quad (17)$$

Is equal to rank 'n'.

A system is controllable when the rank of the matrix A is *n*, and the rank of the controllability matrix is equal to:

$$Rank(Qc) = Rank(A^{-1}Qc) = n \quad (18)$$

Using the MATLAB command **ctrb,** the rank of the matrix and hence the controllability of the quarter vehicle model can be determined.

i.e. M = ctrb(A,B) (19)

the rank is entered as: rank_of_M = rank (M) (20)

4.3 Observability Matrix

Recall equation (5) and (6) for a linear time invariant continuous system, the system is observable if and only if, n x n_m composite matrix is obtained as:

$$Qb = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (21)$$

the system is observable if and only if the Q matrix has a rank of n.

Using the MATLAB command **obsv,** the rank of the matrix and hence the observability of the quarter vehicle model can be determined.

i.e. N = (obsv(A,C))' (22)

rank is entered as: Rank_of_N=rank (N) (23)

where A and C are the system matrix and output matrix respectively.

5. Results and Discussion

Controllability

M =

1.0e+07 *

0	0	0.0000	0.0000	-0.0000	0.0001	-0.0000	0.0028
0.0000	0.0000	-0.0000	0.0001	-0.0000	0.0028	0.0000	-0.1611
0	-0.0000	0.0000	-0.0001	-0.0000	0.0010	-0.0000	0.1050
0.0000	-0.0001	0	0.0000	-0.0000	0.1156	0.0000	-1.1882

rank_of_M =

4

From the results above the rank of the matrix M = 4 which makes the system to be controllable.

Observability

N =

1.0e+04 *

0	0.0000	0.0886	-0.9782
0	0	0.0000	0.0886
0.0001	-0.0011	-0.1188	2.7068
0	0.0001	-0.0011	-0.1188

rank_of_N =

4

From the results above the rank of the matrix N = 4 which makes the system to be observable. This condition indicates that the measurement of the output variable allows the determination of the state. i.e. the states are observable.

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